

# Game Theory Application in Various Fields of Work

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**Abstract:** This research is in context with Game Theory and its various applications in the fields of Finance, International relations, Operations Research and Poker.

The same theory can be applied in politics to explain the crisis between Iran and Israel or in finance to see how an individual makes decisions to satisfy their own objectives based on a firm's dividend decision, or its application in movies such as 'The battle of the sexes' and even in poker.

**Keywords:** Game Theory application, International relations, Operations Research.

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## 1. GAME THEORY

On a day out with friends, one would probably not think too much about the decisions that they're making, but there is an entire field of math and sciences that applies to social interactions. This vast field is known as *game theory*. Mathematician John Nash pioneered the game theory in the 1950s. Gradually this theory has been able to offer various insights into economic, political, gambling and even finance. It has a wide scope as it can apply practically to any situation where people get together and are involved in each other's business.

According to Roger B. Myerson, "Game theory has a general scope, encompassing questions that are basic to all of the social sciences."

As per definition, game theory is the study of mathematical models of conflict and cooperation between intelligent decision-makers. Thus with this theory it is possible to understand the decision making process between countries in political matters, or even by two companies in a competitive business.

The theory is based upon two assumptions that the players involved are *intelligent* and make *rational* decisions. However, being rational in game theory is different from its generally meaning. For example, a nuclear pile-up by all countries seems like an irrational move but in a game it's a rational decision. It is the study of how players determine their strategies, given the tactics of other players in the game.

The game theory has two main branches: Cooperative and Non-Cooperative (Competitive). Cooperative games are where all players involved agree to work together toward a common goal. For example, when a group of nations decide on how to divide up the burden for stopping climate change, or a group of friends deciding how to split the cost for dinner. Non-Cooperative game theory covers competitive social interactions, where there would be some winners and some losers. This theory can be best explained by the classic example of a prisoner's dilemma:

Two crime partners – Sam and John are arrested at a crime scene and with sufficient evidences against them, both are going to spend 3 years in jail. However, both of them are offered a deal: if they confess to the crime and their partner does not, they would be granted immunity and be set free for cooperating. However, their partner would be in jail for 12 years. If both confess to the crime, both will end up spending 6 years in jail. If neither of them confesses, both will spend 3 years in jail. Both Sam and John are split up and have to make their decisions independently.

Of course the best overall outcome would be if neither of them confess, as this way both would spend as little time in prison - 3 years. However both of them, being criminals don't have any special loyalty toward each other and have no

reason to not confess against each other. The option of immunity sounds good to both of them and if one confesses and the other doesn't, the one to confess walks away freely. Sam might consider not confessing, because if John doesn't confess either, they both only serve 3 years. So if they could really trust each other, this would be their best decision. However, Sam cannot be sure that John would not confess, as he would have a lot to gain (immunity) by confessing. So if John confesses and Sam does not confess, he (Sam) would risk 12 years in prison, whereas John would be free. The same may bother John with his decision to not confess. With this, the option for both to confess and spend 6 years in prison would not sound so bad. Thus, this would be game theory's solution, where they would both confess and spend the same number of years in prison (6 years).

The same theory can be applied in politics to explain the crisis between Iran and Israel or in finance to see how an individual makes decisions to satisfy their own objectives based on a firm's dividend decision, or its application in movies such as 'The battle of the sexes' and even in poker.

## 2. GAME THEORY OPTIMAL SOLUTIONS AND POKER: A FEW THOUGHTS

GTO stands for "game theory optimal." In poker, this term gets thrown around to signal a few different concepts. It refers to thoughts about opponent modelling, and thinking about poker situations in terms of ranges and probabilities, as opposed to being strictly results oriented.

Sometimes those ideas gets reduced to young pros shouting across a poker room (or the Twittersphere) about whether a given play is "GTO" — or even "the opposite of GTO," as I recently saw in a discussion thread. But what does this really mean? And does it apply to your game?

### Seeking an Un Exploitable Strategy:

A game theory optimal solution to a game has precise mathematical definitions. It is interesting to consider what this means to a poker player, as well as how this concept has become a dominant framework for looking at ideal poker strategy. Since most of my time these days is spent building computer AIs that play strong poker, it often leaves people thinking about how computers look to GTO strategies for playing an exploitable poker.

GTO — especially in the context of modern poker — is largely about pursuing a strategy that makes it impossible for you to get pushed around. Think Uma Thurman in *Kill Bill*. Or Bruce Willis in any Bruce Willis movie.

Outside of poker, GTO is usually introduced with the "prisoner's dilemma." In this hypothetical situation, the two of us are arrested for jointly committing a crime. If neither of us talks, we both get off with light sentences. However, if one of us snitches on the other, the snitch will get off with no punishment at all, while the person who doesn't talk gets a harsh sentence. If we both snitch, we both get a harsh sentence, since each person's testimony can be used against the other.

Even though we would be best off with the first scenario (nobody talks), each individual is better off from collaborating with the authorities, regardless of what the other does (if I don't snitch, you should snitch to get off free, and if I do snitch, you should definitely snitch as well). In an environment where players are rewarded for taking advantage of each other, it may not be worth acting cooperatively, even if all sides would be better off by doing so.

### Another dilemma: a tournament bubble:

The poker equivalent would be two players fighting it out on the bubble of a tournament. Except for the super-deep stacks who can chip up on the bubble with no risk of busting, the remaining players benefit from any confrontation that leads to an elimination. Thus the two players in the hand are only hurting themselves, by trying to bust each other. And yet, it's not possible for them to collaborate toward a mutually beneficial solution.

Reacting to an opponent's attempts to run you over is so natural to a thinking poker player, framing it in terms of GTO can seem almost superfluous. Of course your opponent has a strategy. You have some idea of what that strategy would be with various hands, and your job is to take that into account when executing your own strategy.

In other words, play the player. This is what GTO is all about.

### The Quest to "Solve" Hold'em (and Other Games):

As you adjust your strategy to an opponent's strategy, he or she will adjust to yours, and so forth. For heads-up limit hold'em, the University of Alberta team took this process to its logical conclusion, publishing their results earlier this

year in Science magazine. Using a network of computers, they set two strategies loose, repeatedly adjusting to each others' play. Eventually, they reached a state where neither player could gain even a 1% advantage against the other in any specific situation.

This sounds complicated, and I'm simplifying what they did slightly. But in essence, they reached a strategy which an opponent cannot exploit — or at least cannot exploit beyond a 1% edge — with any other possible strategy. Somewhat confusingly, the University of Alberta team claims both to have “solved” heads up limit hold'em, and also that they found just one GTO equilibrium for heads-up limit hold'em, and that there are likely to be other equilibria for the game, left to be discovered.

However, the Alberta folks are quick to point out that calling a three-bet with two Aces on the button 100% of the time is only optimal in the GTO equilibrium that they found. Given the rest of their strategy, it would be worse to four-bet with pocket aces. You probably could four-bet with aces, but then the rest of the strategy would need to adjust. At the very least, you'd need to four-bet other hands, too, so as not to give it away that you had aces. If they fixed two Aces as a four-bet and ran the rest of the process until it stabilized, would it reach a different GTO equilibrium? That would be an interesting experiment.

In practice, if you know that your opponent will call off with one-pair hands against aces, and not react as though he knows your very tight four-betting range, then you're just missing a bet. In a recent episode of The Thinking Poker Podcast, **Andrew Brokos** and **Nate Meyvis** explain this point well. Game theory uses a strong definition of optimal play, where you're supposed to consider every play you would ever make with any hand as part of the equilibrium. However in real cases, 95% of that is optimizing for what you would do in this spot, given the range of hands that you could be playing, and what your opponents' hands might be.

In a hand discussed on the show, a listener in a limit hold'em game held out of position on an ace-high flop. You're not getting an ace to fold, and by checking, you'll get more value from a bluff, as well as from a value bet with middle pair.

Let's think about this situation as a computer AI might. Say you're playing \$100/\$200 limit hold'em. The pot is \$400, and you raised preflop. Your hand's value might be something like +\$700 at this point (including the odds of winning the pot, and the value of future bets). Now the ace flops, and your value drops to +\$300 or so. More importantly, the value of check-calling might drop by less than the value of betting out. Estimating the value of your hand, assuming that both players play well and about even in the long run, is just another way of approximating GTO.

#### **When Everyone Knows What Everyone Else Is Doing:**

Once you're in three-handed (or more) games, there is no game theory optimal solution, strictly speaking. This is because there is no stable equilibrium (or too many equilibria to count, depending on whom you ask). The players can always adjust to each other, or take advantage of a player trying to execute a GTO strategy and not adjusting to them, through a process that **Bill Chen** and **Jerrod Ankenman** call “implicit collusion” in their 2006 book *The Mathematics of Poker*. Thus there is no unexploitable strategy.

Let's dig into this for a second. When playing heads up, if you (or a bot) follows a GTO strategy, an opponent can't beat you in the long run, no matter what he or she does. This does not mean that you are winning the most against this opponent, but you are locking in a long-term tie, while still benefitting from some of your opponent's mistakes.

For example, the limit hold'em GTO bot will pay off on the river with bottom pair often enough so you can't bluff it effectively. If you never bluff in this spot, the bot will still pay you off at the same rate. An exploitative player would stop paying you off after a while, and win even more. **Doug Polk** spoke on the TwoPlusTwo Pokercast about this situation coming up during the man-vs.-machine NLH match last spring. It was such a relief to the players once they realized that while the computer played well (“4 out of 10” compared to his regular opponents, according to Polk), it did not attempt to exploit their betting patterns. If when you flop the nuts you bet 1.5x the pot or crumble a cookie, the AI doesn't know or care. It just plays GTO.

In an idealized 3+ player game where everyone adjusts to everyone, GTO should not work. But in practice, if the players don't change their strategies too much from hand to hand (and they don't), a lot of the heads-up GTO principles apply.

A youngster went to graduate school with one of the best online players in the world, and had a chance to watch him play. He was surprised that his classmate did not make any unusual plays, or really any “moves” at all. According to the pro:

- everybody knows who I am
- everybody knows how I play
- there's no reason to get out of line

If you take Chen and Ankenman's ideas about "implicit collusion" to heart, one could also add that if the players were ganging up on him instead of trying to beat each other, the pro would just quit the game. This is a non-issue in the nosebleed games, since everyone knows everyone else, and playing anonymously or collusively isn't really possible.

The point is, the best player in online poker last year (on a per-hand basis) plays GTO. He must be really good at knowing when to bet 80% of the time and to call 20%, and when to call 20% and to fold 80%. And then he actually does it.

### 3. CONCLUSION: GTO IS THE BASELINE

In the short term, the humans are converging on GTO more quickly. When you sit in the stands in the Amazon Room at the Rio All-Suite Hotel and Casino for the final table of the \$1 Million Big One for One Drop, it will shock you how loose-passive a play becomes after getting down to three-handed.

It took some flack on Twitter for comparing the small-bet, check-down game happening between Daniel Negreanu, Dan Colman and Christoph Vogelsang to a nightly satellite at a local casino. Folks fired back that those guys were the best in the world. But it sure did look like none of the players were trying to pressure the others. And why should they? With payouts of \$15M, \$8M, \$4M, there was a lot less upside in winning chips than was the downside to chipping down or busting.

On other hand, if one guy pushed, he knew the others knew how to fight back. So nobody pushed. For about two hours, three of the best short-handed NLHE players in the world checked or small-bet every hand, until Vogelsang, the short stack, busted.

Do you need to play GTO in order to win? Or rather, how close do you need to get to GTO in order to hold your own against a strong set of opponents? Let's let Professor Tuomos Sandholm, head of Carnegie Mellon's Claudico no-limit hold'em team, answer that question.

In a recent article in *Cigar Aficionado* interviewing academics and enthusiasts at the Annual Computer Poker Championship, Sandholm was asked about his colleagues at the University of Alberta solving limit hold'em.

"They say it is essentially solved. I think that counts," responded Sandholm. "My question though is: Was it essentially solved three years ago?"

Near-optimal GTO play is just the first step. Once your baseline strategy can't be easily exploited, you can spend the rest of your time studying opponents' tendencies and adjusting to their weaknesses. There will be plenty of opponents who don't think about ranges, who don't adjust to some of the game information, or who are just playing their own way. Adjusting to them is what GTO, and poker, is really all about.

For a good, accessible exploration of how to use GTO in your game, check out Ed Miller's book *Poker's 1%: The One Big Secret That Keeps Elite Players On Top*. Meanwhile for examples of how players use hand ranges to adjust to their opponents' strategies, see any of Alec Torelli's "Hand of the Day" analyses here on *PokerNews*, or the recent interview with Vegas \$2/\$5 NL pro Sangni Zhao.

#### Game Theory in Operations Research with example "Battle of the Sexes":

Operation research (OR) is an analytical method of solving problems and making decisions that are useful in efficient management of an organisation. In operations research, the problem is broken down into basic part/components and then it is solved in pre defined steps by mathematical analysis.

The analytical methods used in OR consist of math logic, simulation, network analysis, and game theory. This process can be broken down into three steps.

1. A set of alternative solutions to the same problem are derived.
2. The alternatives derived in the first step are analysed, evaluated and then reduced to a smaller set of solutions most likely to prove the most efficient for the given problem.

3. The alternatives derived in the second step are subjected to simulated implementation and, if possible they are tested out in real-world situations.

Game theory is the study of mathematical models of negotiation, conflict and cooperation between individuals, organizations and governments. The study has direct applications in contract theory, economics, sociology and psychology.

Game theory can be used in various areas of study to understand why an individual makes a particular decision and how the decisions made by one individual affect others, whether positively or negatively.

Game theory research involves studies of the interactions with people or groups of people. Because people always make use of an increasing number and a variety of technology to achieve the final results, game theory can also be directly applied in the areas of negotiation, such as contracts and indirectly applied in practical professional studies such as engineering, information technology and computer science.

“Games” can range from simple personal or small group encounters or problems to major confrontations between corporations or superpowers. One of the primary objectives of the game theory is to determine the optimum strategy for dealing with a given situation or confrontation. This can involve goals such as maximising one party’s gains, maximising the probability so that a specific goal can be reached, minimising one's risks or losses, or inflicting the greatest possible damage on adversaries.

The formal application of game theory requires knowledge of the following details: the identity of independent actors, their preferences, what they know, which strategic acts they are allowed to make, and how each decision influences the outcome of the game. Depending on the model, various other requirements or assumptions may be necessary. Finally, each independent actor is assumed to be rational.

**Example: Battle of the sexes:**

In this example, a couple conflicts over what to do during the weekend. Both want to do the same activity as they wish to spend the weekend together, but they cannot agree over what to do. The man prefers to go watch a cricket match, whereas the woman wants to go shopping. This is a classical example of a coordination game, which is a sub part in pure strategies, in Operations Research.

Since the couple wants to spend time together, if they go separate ways, they will receive no satisfaction and hence the set of payoffs will be 0, 0. If they go either shopping or to a cricket match, both will receive some utility from the fact that they’re together, but one of them will actually enjoy the activity. The description of this game in strategic form is therefore as follows and using the sigma equations to find the probability:

	WOMAN		
	Activity	Shopping(2/3)	Cricket(1/3)
MAN	Shopping(1/3)	1,2 2/9	0,0 1/9
	Cricket(2/3)	0,0 4/9	2,1 2/9

Using Nash Equilibrium

For Man:

$$1*(2/9) + 0+0+2*(2/9) = 2/3$$

For Woman

$$1*(2/9) + 0+0+2*(2/9) = 2/3$$

This shows that the utility for the man or woman doing the same activity is the same. Hence, in this case they can randomly decide what to do. And this can be decided by just flipping a coin as well.

#### 4. THE APPLICATION OF GAME THEORY IN FINANCE

Objective: to demonstrated is the use of the availability of imperfect information about the health of the firm to aid an individual to make decisions based on their objectives using game theory, based on our understanding of the Miller Modigliani Model for dividend decisions.

In finance one of the important branches is corporate finance which is concerned with the financial decisions a firm makes. Traditionally, theories such as the Miller Modigliani Theory, predominantly used for capital structure decisions and dividend payouts, assumes perfect markets, perfect information and no taxes, an assumption far from what is actually practiced in markets.

Game theory comes into play when traditional theories provide an unsatisfactory explanation for recurring events in firms, by allowing theorist to use data such as strategic interaction to be used during analysis.

In the recent years, game theory has been successful in areas such as explaining why *dividends act as a signal*.

The Miller – Modigliani Theory suggested that dividend decisions signal the health of a firm. This conclusion was then taken forward by using game theory to explain the same. In the general sense of the application of game theory in dividend signaling roots back to the firm’s health being signaled by the dividend decisions, which was demonstrated by Bhattacharya’s 1979 paper. He argued that dividends show how profitable a firm is, by the management using dividends as a tool to demonstrate that the firm is profitable to the extent of declaring dividends, hoping to drive the price of the firm’s stock upwards.

In our game, there are 2 players, the managers, and the shareholders. The managers have perfect information – regarding the true value of the firm, the shareholder’s use signals to determine the same. It is also assumed that the tax on cash dividends is more than the tax on capital gains. Dividend decision have a twofold path- the decision to give out dividends, and the choice of how much. Once the management does disclose the amount of dividend (if any) shareholder react by keeping, selling, or buying additional shares, maximizing their expected utility from each share. In a recent Corporate Finance class that we took, the reaction of the shareholders seemed to have based on the motive of the shareholder (i) growth or (ii) a steady income. Based on that, we created a game theory table – again with 2 players, and their moves.

An Individual Shareholder’s use of game theory for decision making.

Shareholders	Managers	
	Distribute Dividends	Don’t Distribute Dividends
Growth	Buy shares, Market Volatility	Keep Shares
Steady Income	Keep Shares	Sell Shares, Market Volatility

The signals: If dividends are distributed, the per share price is less than if dividends are not. (This is once the market settles, and is based on MM theory where  $P1=PO(1+Ke)+D1$  )

The outcome: if dividends are distributed, for a shareholder looking for growth, they should buy additional shares, to receive dividends on the same.

If dividends are not distributed, the shareholder looking for a steady income should sell their shares, as the market reaction to the same would increase the share price, leading to a capital gain for the investor.

Game theory is a logical tool used by rational decision makers, which can be applied in finance both on an individual level (as demonstrated) and, like several studies and papers suggest, at a corporate decision making level.

#### **To see the use of game of theory in politics we use the Iran Israel crises as an example-**

Iran’s nuclear research program generates global concern and suspicion. The general idea is that if Iran continues its research, and, say, an Israeli attack occurs, Iran can retaliate. Thus, any potential attacker has to foresee the consequences of its action against Iran. Iran then has to assess the magnitude of costs that deter a potential attacker. Yet, the attacker can guess that Iran has strong incentives to misrepresent its incentives in its will and ability to retaliate. Iran can in turn try to guess whether the attacker takes its stand about retaliation seriously.

We can qualify the above interaction as constituting a game, ie a situation of strategy interdependence.

Take for example the problem of Iran’s nuclear facilities program constitutes yet another source of friction between Iran and Israel. To see how events will pan out we resort to a simple 2x2 model to depict how game theory would be applied in this case. Assume Israel has two strategies, attack or do not attack, we also assume Iran has two strategies stop nuclear research or continue with its nuclear research. Hence we have two players with two strategies each. We now make an outcome matrix.

		Israel	
		Attack	Do not attack
Iran	Stop	Outcome 1	Outcome 2
	Continue	Outcome 3	Outcome 4

To obtain a game matrix we need to specify both countries preferences over these outcomes.

Let’s assume that Iran main objective is to become a nuclear power while Israel’s is inverse. Supposing an Israeli attack cannot destroy all the Iranian facilities Iran would most likely prefer outcomes 3 and 4 over 1 and 2. The decision “stop” prevents Iran from obtaining its primary objective i.e. to become a nuclear power. Thus, for Iran, we have {outcome 3, outcome 4} > {outcome 1, outcome 2}.

Suppose also that Iran prefers outcome 4 to outcome 3 and outcome 2 to outcome 1 as it prefers no Israeli attack; its secondary objective. These assumptions generate the following preference ordering for Iran: outcome 4 > outcome 3 > outcome 2 > outcome 1.

Israel on the other hand prefers outcomes 1,2 over 3,4 as Iran’s decision to stop realizes Israel’s main objective ie to stop Iran from becoming a nuclear power. Thus, for Israel, we have {outcome 1, outcome 2} > {outcome 3, outcome 4}. Suppose Israel prefers outcome 2 to outcome 1 and outcome 4 to outcome 3 as it prefers to avoid a military failure. These assumptions generate the following preference ordering for Israel: outcome 2 > outcome 1 > outcome 4 > outcome 3. Now assume also ordinal-level preferences, with 4 indicating the best, 3 the next-best, 2 the next-worst, and 1 the worst outcome for players.

		Israel	
		Attack	Do not attack
Iran	Stop	1,3	2,4
	continue	3,1	4,2

The first number in each cell denotes Iran’s preferences for that outcome while the second depicts that of Israel.

We realize Israel obtains better outcomes by choosing ‘do not attack’ regardless of Iran’s choices. Israel obtains 4 instead of 3 against Iran’s decision to ‘stop’ and 2 instead of 1 instead of Iran’s decision to ‘continue. Similarly, Iran obtains better outcomes by choosing “continue” regardless Israeli choices: Iran obtains 3 instead of 1 against Israeli decision “attack” and 4 instead of 2 against Israeli decision of “do not attack”. Therefore the equilibrium is ‘continue, do not attack’. Thus we explain the current status quo between Israel and Iran regarding Iranian nuclear research through simplification. there are only two players, each has two strategies, they simultaneously interact only once, their preferences are ordered according to primary and secondary objectives, and each player strives to obtain highest possible outcome given other’s choices.

### 5. CONCLUSION

Thus, except for the already mentioned, game theory has wide ranging application through all fields, including Mathematics, Computer science, Biology, Psychology, Law just to name a few. It is a growing study and many schools and colleges have begun to teach it to their students, including online courses offered by *University of Yale* and *Stanford*. There have been a number of books written on game theory, that of *Philip J. Davis (1970)*, *Roger B. Myerson (1991)* and *Steven Tadelis (2013)*. This field has vast scope and could be used in almost any subject, as long as there are two individuals who are involved in each other’s business.

#### REFERENCES

- [1] <http://www.investopedia.com/ask/answers/09/game-theory-business.asp>
- [2] <https://m.youtube.com/watch?v=MHS-hjtGgSY>
- [3] <https://books.google.co.in/books?hl=en&lr=&id=22eObsr-cUQC&oi=fnd&pg=PR7&dq=game+theory+defined&ots=xcilvoqNwT&sig=CxbO1HPvLeC68Cudf41oVVNz5fo#v=onepage&q=game%20theory%20defined&f=false>
- [4] <http://www.cardplayer.com/poker-news/17562-poker-strategy-with-reid-young-game-theory-vs-pot-odds>
- [5] <http://www.doylebrunson.com/education/intermediate-lessons/lesson8/>
- [6] <http://www.universalteachpublications.com/univ/ebooks/or/Ch9/strategy.htm>
- [7] <https://www.slideshare.net/mobile/benghuid/game-theory-6705811>
- [8] <http://gametheory101.com/courses/game-theory-101/battle-of-the-sexes/>
- [9] <https://sites.google.com/site/gameoptprakrita/bos>
- [10] <http://meissereconomics.com/2016/06/01/Boskalis.html>
- [11] [https://books.google.co.in/books?hl=en&lr=&id=jnz0BwAAQBAJ&oi=fnd&pg=PA1&dq=game+theory+dividend&ots=mM3ZNtmsNV&sig=eh-MVLavJ\\_JKc6Wk6wrNAk5EEyc#v=onepage&q=game%20theory%20dividend&f=false](https://books.google.co.in/books?hl=en&lr=&id=jnz0BwAAQBAJ&oi=fnd&pg=PA1&dq=game+theory+dividend&ots=mM3ZNtmsNV&sig=eh-MVLavJ_JKc6Wk6wrNAk5EEyc#v=onepage&q=game%20theory%20dividend&f=false)
- [12] <http://www.e-ir.info/2012/06/21/a-short-note-on-the-use-of-game-theory-in-analyses-of-international-relations/>